IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 7 Issue 12, December 2020
ISSN (Online) 2348 – 7968 | Impact Factor (2020) – 6.72
www.ijiset.com

Flood Forecasting using the Avenue of Models

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Abstract

Flood forecasting is only part of more comprehensive water resources management activities which relate to flood warning, flood control or reservoir operation. This paper focuses on the computational aspect of flood forecasting on middle sized catchments. Flood forecasting is a daunting issue in hydrology. Structural solutions are mainly preventative and focusing on curtailing the magnitude of floods using different methods such as dams, embankment, compound channels, widening of river beds, etc. However these solutions have adverse environmental, hydrologic, ecologic or economic consequences. The non-structural mitigating measure places people away from flood. This method is designed to reduce the impact of flooding to society and economy. Rainfall runoff modeling for the flood forecasting and warning schemes is a non-structural hydrologic method for mitigating flood damages. The relationship between rainfall and runoff is an important parameter for flood forecasting.

Keywords

Flood forecasting, stream flow, MINIMAX model, Data-driven fF models, MIKE 11 Nam

Introduction

Flood forecasting is the use of forecasted precipitation and stream flow data in rainfall-runoff and stream flow routing models to forecast flow rates and water levels for periods ranging from a few hours to days ahead, depending on the size of the watershed or river basin. Flood forecasting can also make use of forecasts of precipitation in an attempt to extend the lead-time available. Flood forecasting is an important component of flood warning, where the distinction between the two is that the outcome of flood forecasting is a set of forecast time-profiles of channel flows or river levels at various locations, while "flood warning" is the task of making use of these forecasts to tell decisions on warnings of floods. Real-time flood forecasting at regional area can be done within seconds by using the technology of artificial neural network. Effective real-time flood forecasting models could be useful for early warning and disaster prevention.

THE FLOOD FORECASTING SYSTEM

• Météorological forecasting

• Rainfall forecasting, nowcasting

• Quantification and spatialization of rainfalls

• Discharges/water stages forecasting

• Flood plains forecasting

• Message and delivery service/system

• Evaluation of the system

According to the various concepts used in developing models, the models can be classified into five categories -

a) Based on correlation/coaxial diagrams between two variables or even more;

b) Mathematical equations developed using regression/multiple linear regression techniques which combines independent variable with one or more than one variable;

c) Hydrological models

c.1 Rainfall run-off model

i) Lumped

ii) Quasi-distributed

iii) Distributed

c.2 Routing techniques

i) Lumped, & Distributed

d) Hydraulic models

i) Dynamic Wave routing

e) Data driven hydrological models

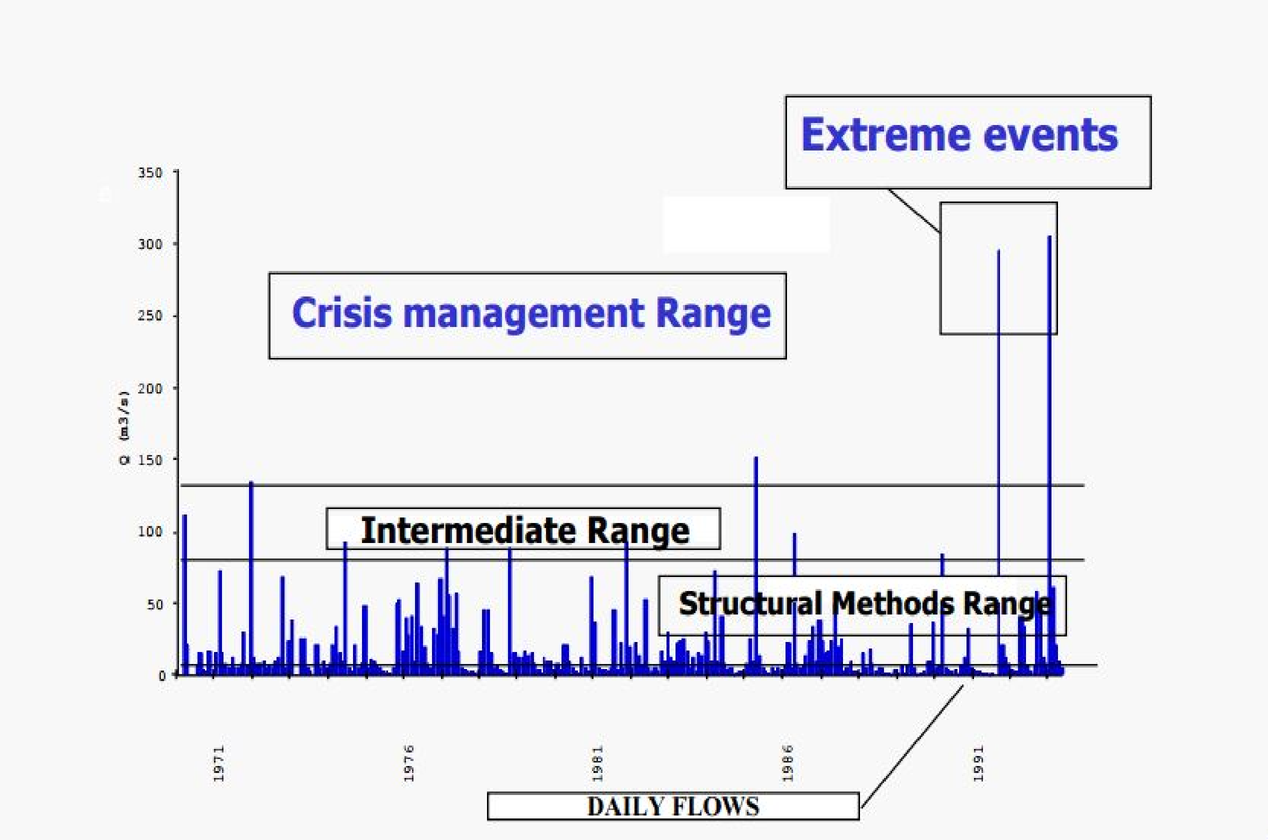
i) Artificial Neural Networks

ii) Fuzzy expert system design for FF

iii) ANFIS (Adaptive Neuro-Fuzzy Inference System) models

The MINIMAX MODEL

There is a real danger in only minimizing the “maximum” (extreme floods) without really improving the global system and the medium or moderate floods. However, and because of the media coverage, this is the actual trend - To find a good compromise between Prevention and Prevision is a real challenge. Flood forecasting and warning have to be incorporated within the global flood management scheme.



FLOOD/WARNING SYSTEMS ARE USERS ORIENTED

The USERS are both :

For the scientists, the users are technical Services and institutional bodies : They operate the networks, our models, … (all technical systems), and design and deliver appropriate messages to End users (ie individuals, citizens, …, nothing to do with technical people) who are the recipients of the messages (dissemination response) and whose comfort, quality of life ,and sometime just life, depend of them.

- If the research and technical object to consider is the global warning system, who is in charge of the this global picture ?
- If we consider only the hydrological/meteorological part of the system, what is the maximum possible benefit for the global system, due to improvements of these “hydromet” components, and who is in charge of the global evaluation, to draw future priorities for research and transfer ?
- Hydromet components : Robustness or accuracy ?

Flood Forecasts, Lead Time And Time To Peak

Forecasts at downstream site derived from :

- Observations/measurements of water stages at downstream and upstream sites
- Measurements of (catchments) and upstream reach flows
- Observation of rainfall (snow melting)
- Forecasts of rainfall
- Nowcasting and forecasts from other basins

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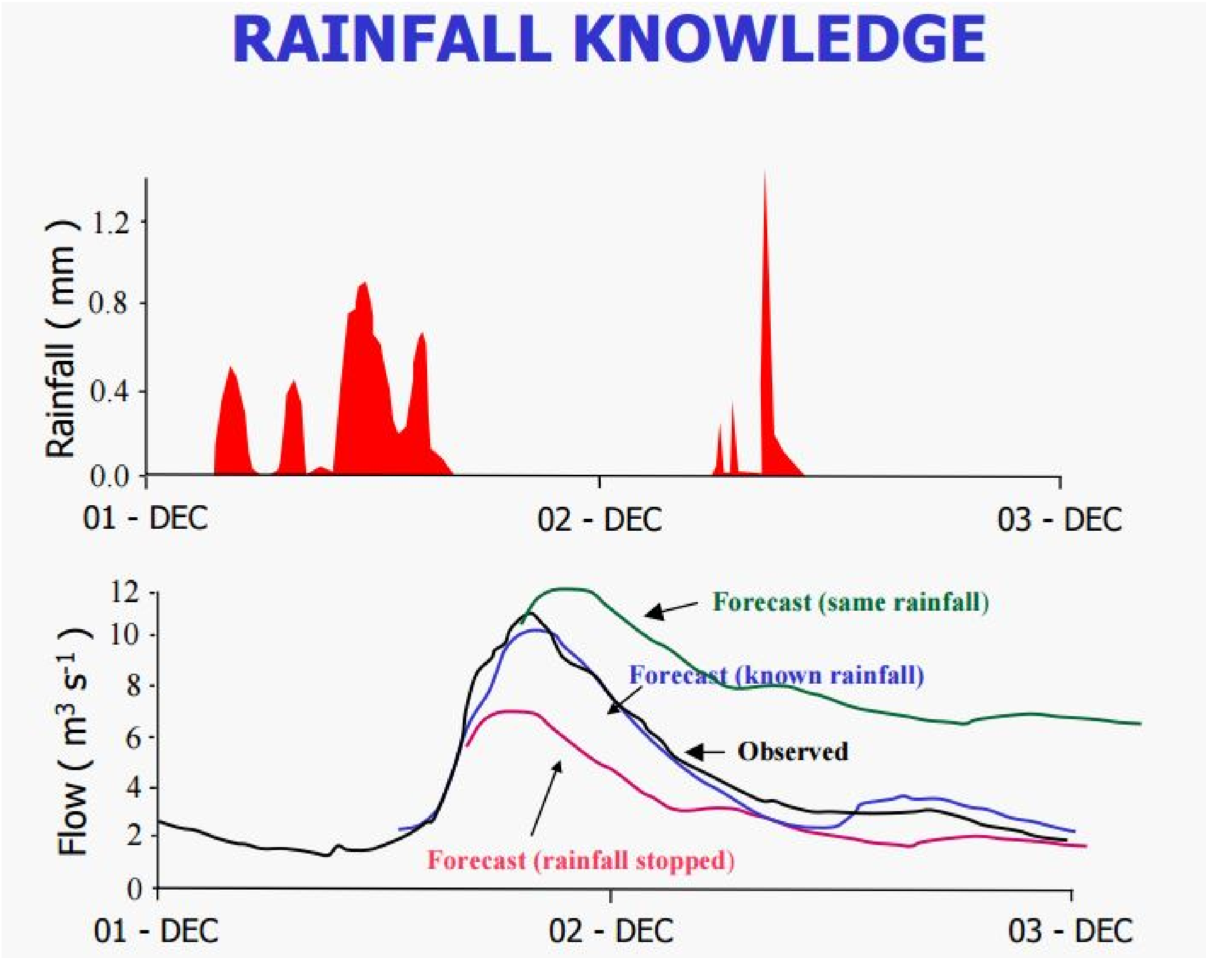
TENTATIVE S/T LOCKS AND CRITICAL COMPONENTS IDENTIFICATION

- For the headwater catchments, the nowcasting, the rainfall observations and forecasts, the snow melting modelling are, at least, critical components.
- For the downstream forecasting sites, flow routing in main channels (including upstream reaches) are critical components.
- At basin scale, the improvement of runoff production (robust across scale) and water transfers controlled by topography and soil is important. All spatialized relevant information (geology, terrain, land use and land cover - DOT) is also important, and the incorporation of embedded meteorological information at different scales as well.

RAINFALL OBSERVATION AND MEASUREMENTS

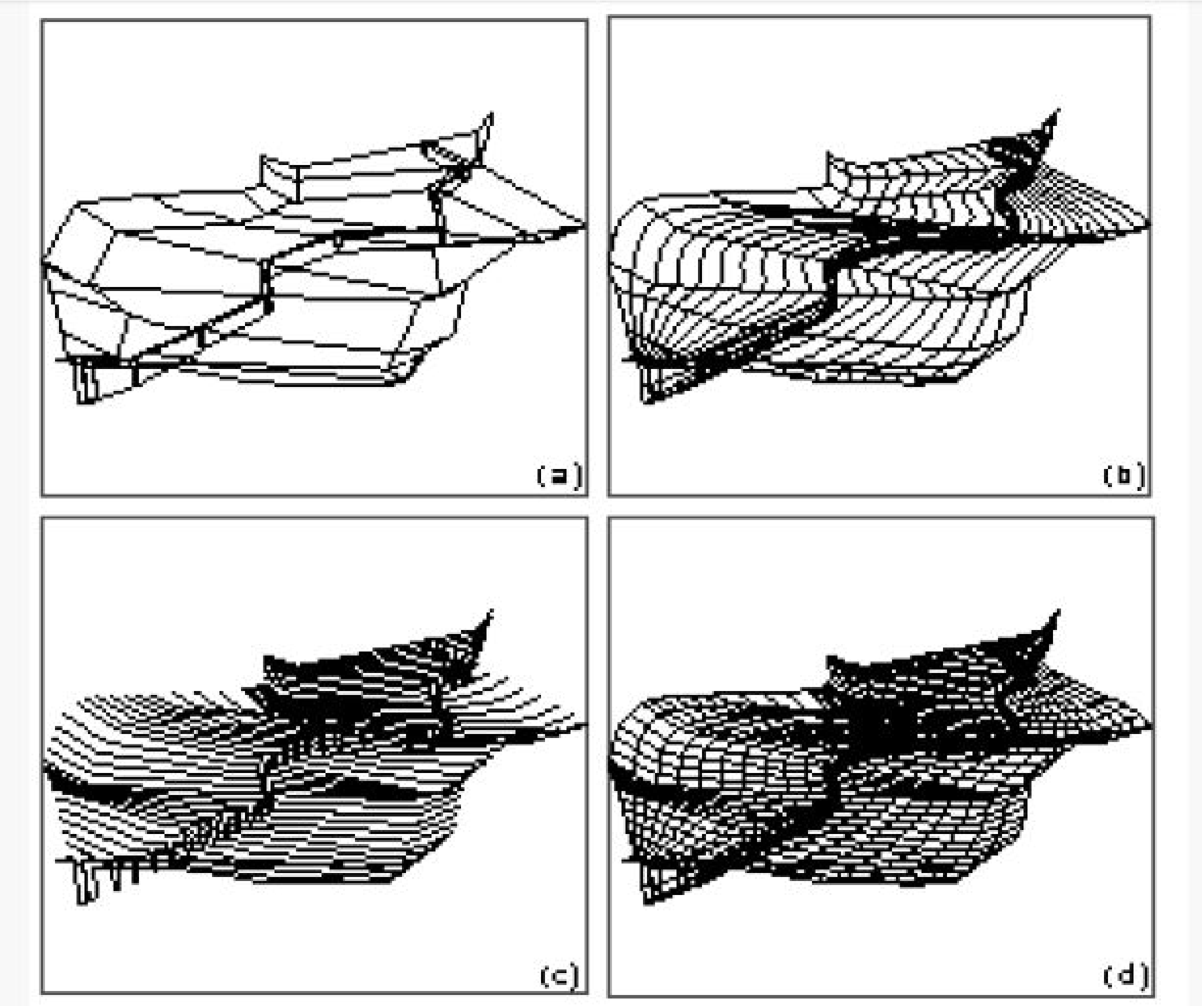
It’s now obvious that hydrological radar will be soon the most appropriate device in rainfall measurements, in cooperation with the ground systems. The capability to feed hydrological models with quantified spatially consistent rainfall is critical, even for global rainfall-runoff models, and not only for distributed physically based models (robustness and uncertainty ?). That doesn’t mean that only distributed physically based models are relevant modelling components.



ROUTING and INUNDATION MODELLING

In main rivers, the channel flow routing is critical to improve the forecasts. One robust possibility is to use Kinematic wave based models (with lateral flows) as the CEH KW model for example. The difficulty is in using the results of these models to draw the flood plains according to the inundation modelling needs. The hydrodynamics based models are more efficient, but less robust and, to a certain extend, data driven. The 1D hydrodynamics models are robust enough to be used as a component incorporated in operational systems. The main questions are about the needs in geometric data to feed the models, and the operational staff skills in operating them. The flood plains mapping is much more easy from these models if, and only if, only one geometric database is used. The geometric characteristics of the basins are more and more needed in hydrological modelling, generally speaking, the use of DOT is one way of improvements.

1D HYDRAULICS/GEOMATICS MODELLING –



RUNOFF PRODUCTION MODELLING IMPROVEMENTS vs ROUTING IMPROVEMENTS

Improved process representation relevant to scales beyond the point : hillslope, grid, catchment, …, using with a better "yield" the available DTM and DEM, Water tracking on slopes, runoff-production is dominated by sub-horizontal water transfers controlled by topography and soil, Better use of spatial dataset support : terrain, soil, geology, land use, land cover, weather variables.

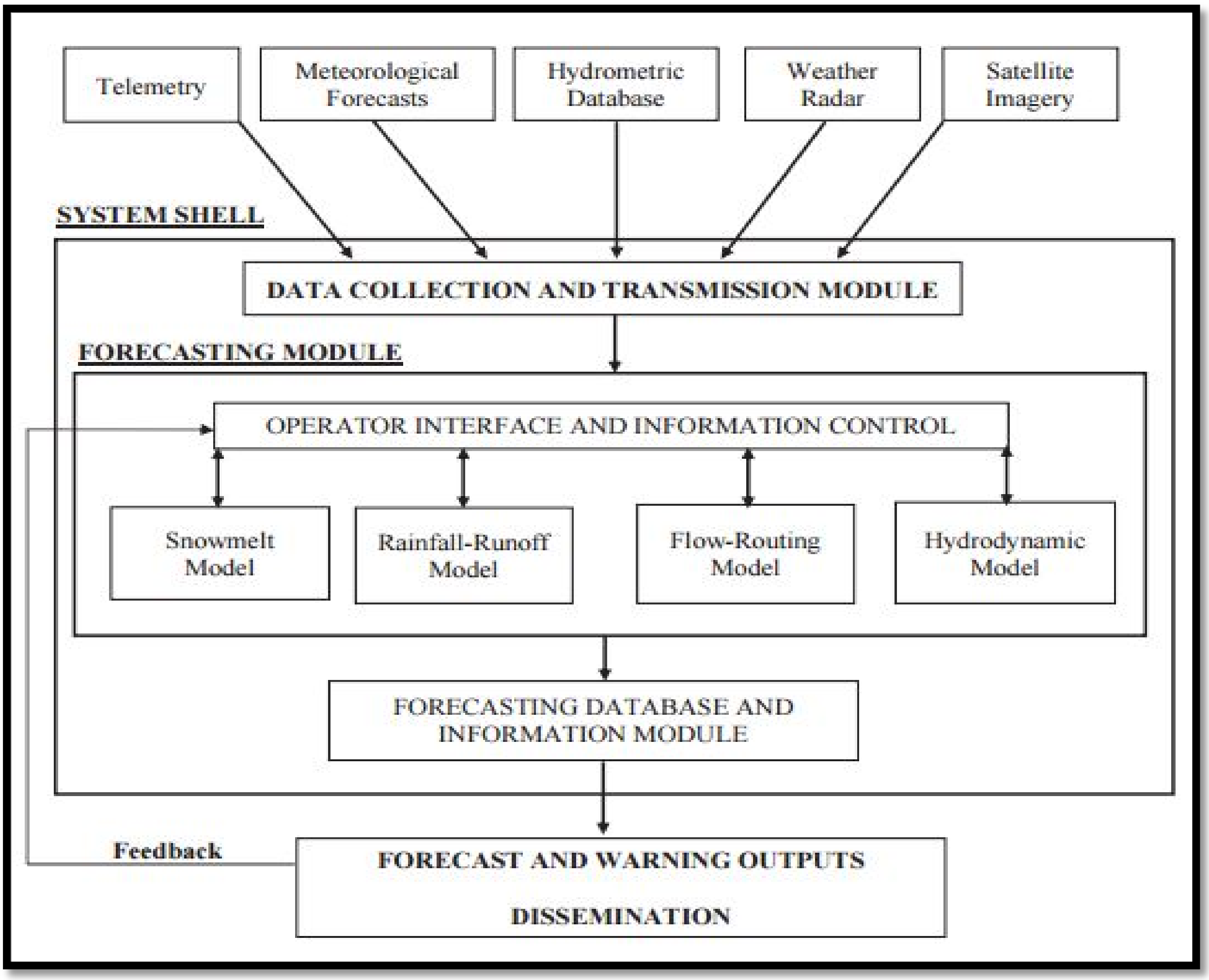
- Improved model transfer to ungauged catchments across scales.
- Whole catchment models, linking rainfall-runoff and hydrodynamic river models.
- Atmospheric/hydrological model coupling

RISK ASSESSMENT

All the technical components have the unique purpose to provide the Service in charge of public warning with relevant information. Messages design and dissemination, and dissemination response, are the most important tasks of a flood forecasting system which must be first a flood warning system The answer is not in hydrological modelling.

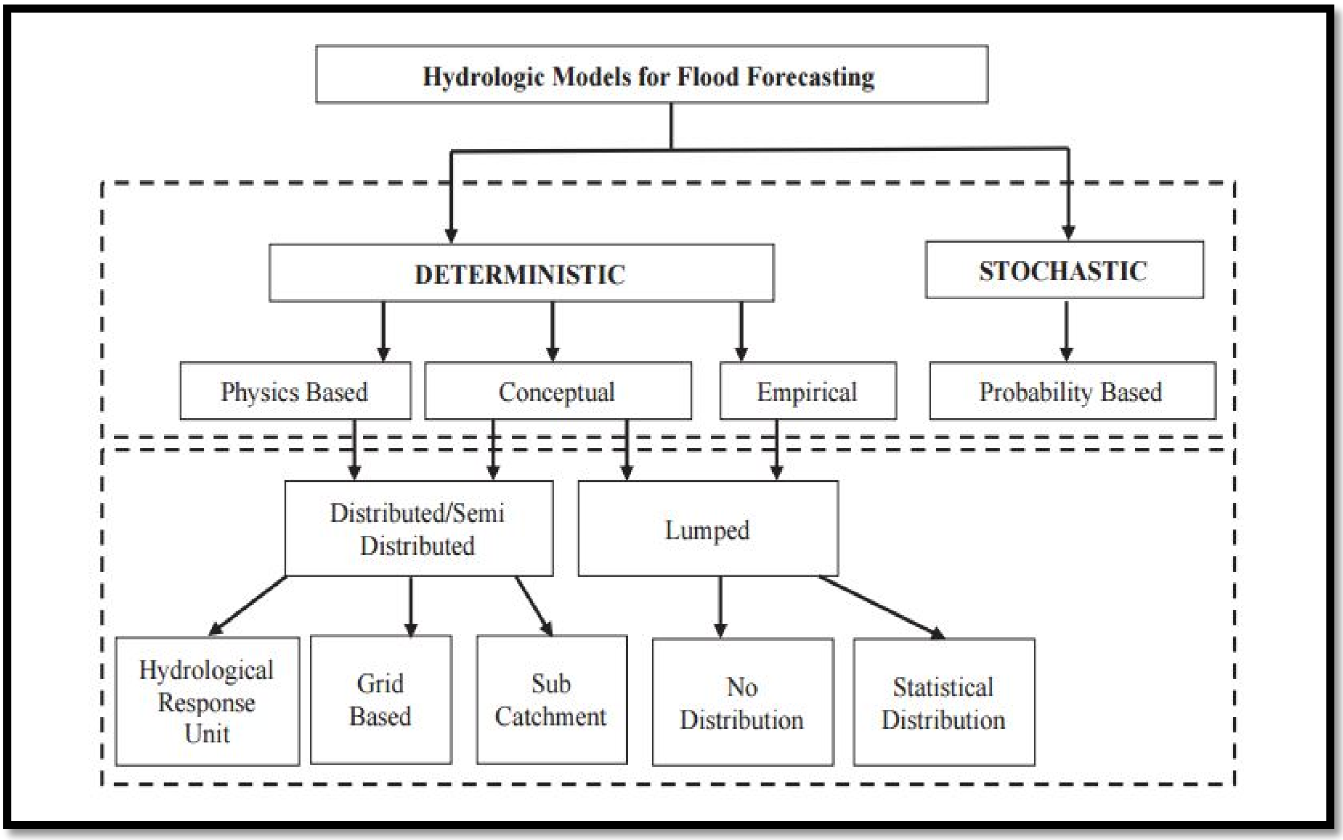
Elements of a flood forecasting (ff) system

The purpose of a flood forecasting and warning system (FFWS) is to alert the general public and concerned authorities of an impending flood as much in advance, andwith as much reliability, as possible. The main components of an FFWS include: (i) data (hydrological, meteorological) collection and transmission; (ii) forecasting, which involves analysis of observations as well as prediction of future rainfall, water elevations and discharge for periods varying from a few hours to a few days ahead; and (iii) dissemination of information to user agencies and communities. Among the various products, the most useful outputs of FFWS are river elevations, inundation extent, and time of occurrence for peak discharges with lead times that are sufficient to initiate appropriate responses by authorities and affected populations. Lead time refers to the period of time between the issue time of the forecast and the beginning of the forecast validity period (WMO 2000). The lead time depends upon the catchment lag time, which varies with basin size and characteristics, as well as characteristics of the storm event. For smaller catchments, especially in mountainous regions where flash floods, associated with the meteorological phenomenon, dam failures, rapid snow melt, ice jams etc., occur frequently, the catchment lag is very small (i.e. minutes to hours). In such areas, including only the rainfall forecast in FFWS may not always improve the utility of FF to users and thus a customized approach may be required (Doswell et al. 1996, Hapuarachchi et al. 2011). For larger basins where catchment lag time is long, an effective lead time can vary from hours to days, and inclusion of rainfall forecast is essential to enhancing the lead time. The factors that impact lead time of forecast in the design of FFWS for a catchment include topographic and hydro-meteorological features of the basin, the dynamics of basin response, and the availability of data. Furthermore, limitations on the level of services (how frequently forecasts are issued and updated, reliability, etc.) are largely dictated by the cost of data collection, modelling constraints, trained professionals, FFWS infrastructure, trans boundary issues, and institutional factors.



Catchment models for flood forecasting

The catchment models used for flood forecasting may be classified according to many criteria. Models may be classified depending upon the way catchment processes are represented – deterministic or data driven; or the way the catchment is spatial discretized – lumped or distributed. Deterministic models solve a set of equations representing the different watershed processes that produce a single model output for a given set of parameters. In contrast, data-driven models provide the capability to simulate the random and probabilistic nature of inputs and responses that govern river flows. The spatial distribution of inputs and parameters is also an important aspect of model selection. In lumped models, the catchment is conceptualized as consisting of various storage tanks representing water storage on the catchment surface, in the root zone, unsaturated zone and in the groundwater zone. Modelling essentially consists of a set of expressions that describe the movement of water through these tanks. The division of precipitation into various compartments is controlled by catchment properties, which are represented in a model by parameters that are tuned during model calibration. In a distributed model, the catchment is divided into a large number of cells or hydrologic response units. While distributed models are generally expected to reproduce the hydrological processes in spatially-varied catchments more accurately, uncertainty in model parameters can lead to substantial errors in distributed models (Carpenter and Georgakakos 2006). Further classification of catchment models is based on the rainfall estimates for lead time, sometimes called the look back window. A model may be an updating or non-updating model. Forecast updating involves the use of the most recent exogenous inputs, such as observed rainfall and observed flows up to and including the time of forecast, to adjust model-computed flows. Many updating models also update Quantitative Precipitation Forecasts (QPFs) within the lead time as observed precipitation data become available. The catchment models based on updating of QPFs are comparatively more accurate and reliable because they use real-time data observed during the lead time.



Data-driven fF models

Data-driven models are often referred to as black-box model because they depend upon the statistical or cause–effect relationships between hydrologic variables without considering the physical processes that underlie the relationships (Luchetta and Manetti, 2003). Data-driven models can include stochastic models (e.g. Regression models, TimeSeries models, and Bayesian models) and nonlinear timeseries models (e.g. Artificial Neural Network models, Fuzzy Systems, and adaptive neural Fuzzy Inference Systems) that require extensive and high-quality time series of hydrologic data. Stochastic models (Box et al. 2016) reflect techniques based on time-series analysis, which have become very popular in hydrology. Stationary stochastic models such as AutoRegressive Moving Average (ARMA) and non-stationary models such as Auto-Regressive Integrated Moving Average (ARIMA) can provide adequate representation of the dynamics of the RR process at large timescales, say monthly or seasonal; parameters of these models have some physical interpretation in those cases. The success of these models can be attributed mainly to their simple mathematics, small computational requirements and their ability to reliably reproduce hydrographs. In the context of operational flood forecasting, ARMA models are mainly used for error correction. Nonlinear time-series models such as Artificial Neural Networks (ANNs) are another example of a data-driven FF approach that can be effective at modelling rainfall-runoff processes and floods forecasting (ASCE Task Committee, 2000a, b). ANNs are nonparametric models that adapt to information inputs and are capable of representing complex nonlinear relationships (Antar et al. 2006, De

Vos and Rientjes 2005). ANNs can learn from input data, generalize behaviour of data, and cope with noise. Comprehensive reviews on applications of ANNs in hydrology can be found in ASCE Task Committee (2000a, b), Abrahart et al. (2010), and Maier et al. (2010). An interesting application of data-driven techniques is to improve the real-time forecasts issued by deterministic lumped RR models, in which the catchment response is simulated by a conceptual model and the residuals are simulated by an ARMA model. Brath and Toth (2002) found substantial improvements in discharge forecasts by coupling ARIMA models with data-driven models (i.e. ANNs) for rainfall forecasting and discharge updating. Despite many successful applications, ANNs have not been deployed in operational flood warning systems, except a few prototype working examples (e.g. Kneale et al. 2001). This can be attributed to various practical issues: long training times, the potential to overfit the model to a dataset, phase-shift errors, and a lack of guidance on architecture and parameter selection (Dawson et al. 2006). Often, ANNmodel-based forecasts are reliable only at small lead times (e.g. one step ahead), which creates uncertainty in their applications for flood management (Prakash et al. 2014). In view of the concerns about the performance of ANNs for FF, it will be helpful to deploy these along with other models in a few pilot applications and evaluate their performance. Another class of data-driven models is based on fuzzy logic and fuzzy set theory (Zadeh 1965). Fuzzy models operate on an IF–THEN principle, where ‘IF’ is a vector of fuzzy explanatory variables and ‘THEN’ of fuzzy consequences (Shrestha et al. 1996). Several approaches have been used to apply fuzzy set theory to flood forecasting, including fuzzy optimization techniques, fuzzy-rule based systems, and combinations of the fuzzy approach with other techniques (Dubrovin et al. 2002). Luchetta and Manetti (2003) compared a fuzzy-logic-based algorithm for hydrologic forecasting to an ANN model and showed that the fuzzy approach outperformed the ANNs. Liong et al. (2000) predicted daily river water levels in the Buriganga River, Bangladesh by using a fuzzy logic model in which the upstream water levels were the inputs. Dubrovin et al. (2002) introduced a model called Fuzzy-State Stochastic Dynamic Programming, which can take into account both uncertainties due to random nature of hydrologic variables and imprecision due to variable discretization. Yu and Chen (2005) proposed an error prediction fuzzy-rule-based method as an updating technique to improve real-time flood forecasts with one to four hours of lead time. The hybrid adaptive neural-based fuzzy inference system (ANFIS) combines ANN and fuzzy theories. Bae et al. (2007) developed an ANFIS-based operational forecasting model for monthly reservoir inflow forecasts using rainfall, inflow, temperature, relative humidity and monthly weather forecasts. Firat et al. (2007) used ANFIS to forecast daily river flows using antecedent flows on the Great Menderes River in Turkey. Data-driven models represent the statistical properties of the system and the relationships between cause and effect variables, but do not represent the underlying physics (Abrahart et al. 2008). Practical applications of the data-driven models for flood forecasting are still lacking chiefly due to the two reasons: (i) data-driven models do not account for the changing dynamics in the physics of the basin over time (i.e. aggregation/ disaggregation/ changing land pattern); and (ii) the parameters of data-driven models are completely dependent on the range of the data (i.e. maximum and minimum) used for calibration. As a result, process-based hydrological models have traditionally dominated FF.

MIKE 11 Nam

Various model have been developed to solve the rainfall runoff relationship in engineering research and practices. The widely known rainfall runoff models identified are the Rational Method (McPherson, 1969), Soil Conservation Service-Curve Number Method (Maidment, 1993), and Green and Ampt Method (Green, 1911). The more complex models which should provide better runoff estimation are continuously being researched and developed. Some of the complex models identified are Genetic Danish MIKE11 NAM (1972). The choice and validity of the model depends on the type of problem, the data availability and the decision to be made. MIKE11 NAM model was applied primarily because of its ability to simulate the watershade physical processes in more detail. MIKE11 NAM model is a watershed lumped-parameter model which are highly relevant with this particular watershed under study and the long term flow simulation desired. In addition, it is a complete and effective modeling software with the adds-on module which allows flexibility for future investigation. The MIKE11 was applied because of its availability in the Hydraulic and Hydrology Department, Universiti Teknologi Malaysia. MIKE11 NAM is a professional engineering software package developed by Danish Hydraulic Institute, Denmark. This one-dimensional modeling tool developed since 1972 has been accepted worldwide especially for water resources, water quality planning and management applications (DHI). Specifically the MIKE 11 software is meant for simulation of flows, water quality and sediment transport in estuaries, rivers, irrigation systems, channels and other water bodies. The MIKE11 NAM, the watershed lumped and conceptual rainfall-runoff model regarded watershed as one unit and the conceptual model are based on considerations of the physical processes (Mike 11 User Manual).

Data Requirements for the MIKE11 NAM model consist of

i) Setup parameters – catchment area, topography and soil properties.

ii) Model parameters – time constants and threshold values for routing of overland flow, interflow and baseflow.

iii) Meterological data – precipitation and potential evaporation.

iv) Streamflow data fpr the model calibration.

The reliability of the MIKE11 NAM was evaluated based on the Efficiency Index (EI) as described by Nash and Sutcliffe (1970). There were several related studies available for model performance evaluation such as by Aitken (1973) and Fleming (1975). The procedure by Nash and Sutcliffe (1970) had been widely used for the detection of systematic errors with respect to long term simulation. The EI was developed to evaluate the percentage of accuracy or goodness of the simulated values with respect to their observed values.

XAJ Model

It is a rainfall-runoff watershed model and is highly suited to the hydrological simulation and forecasting in humid and semi-humid regions. The core of the XAJ model is the concept of saturation excess runoff generation mechanism which describes the runoff generation processes usually happen in humid and semi-humid regions. The XAJ model is a semi-distributed conceptual model. It separates the whole watershed into several sub-basins according to the rainfall stations and Thiessen polygon method. The evapotranspiration, runoff generation, and flow concentration computations are carried out for each sub-basin. The generated runoff is separated into three components, which includes surface runoff, interflow, and groundwater, and then concentrated using multiple linear reservoirs and lag-and-route method, respectively. The above mentioned computations generate the discharge of each sub-basin. The discharges of sub-basins are then routed down the river channels to the whole watershed outlet using the Muskingam successive routing method.

Fuzzy expert system design for flood forecasting

Linguistic terms are chosen to describe the input variable stage and the results. Further refinement of the models could not be achieved by adding extra membership functions. Gaussian membership functions (the function is generally suited for Indian rivers) can be used. Applying a similar method of data classification, membership functions are determined for the output variable discharge.

Rule definition

Some years of average hourlystage data and expert knowledge are used to create a rule base for the fuzzy logic model. Rules are defined for both the high and low extreme conditions, with regard to actual occurrences, because of the physical nature of the relationships. Depending on number of membership functions for each input variable; the minimum rule base is created. For each data point, all rules are evaluated.

Fuzzy model construction

The platform selected for the fuzzy logic expert system is MATLAB and MATLAB'S Fuzzy Logic Toolbox. The variables are combined into rules using the concept of 'AND'. The fuzzy operator 'minimum' is applied as the 'AND' function to combine the variables. No weightings are applied, which means no rule is emphasized as more important in respect to estimating the discharge. Implication is performed with the minimum function, and aggregation is performed with the maximum function. The centre of gravity method is applied as a means of defuzzification of the output membership functions to determine a crisp set. Based on this structure a baseline model fuzzy logic expert system for stage-discharge relationship is constructed for the G&D stations. Alternate functions for the expert system are investigated through sensitivity analysis.

Sensitivity analysis

A sensitivity analysis is performed for the fuzzy logic operator AND, and for methods of implication, aggregation and defuzzification. The results of changing a single operator or method while the rest of the model is held constant are compared with the results from the baseline model. The results are evaluated on the basis of correct linguistic matches. Based on this sensitivity analysis, the AND operator 'minimum' and the implication method 'minimum' are found to perform better than the product method. The fuzzy logic and ANN models are evaluated based on their ability to predict the discharge.

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